Statewide Dual Credit for College Algebra (MATH 1130)
(Advanced Algebra and Trigonometry #3124)

Forty-seven high schools from across Tennessee are participating in the statewide College Algebra dual credit pilot for the 2013-14 school year. As one of the pilot sites, your Advanced Algebra/Trigonometry students have the opportunity to earn College Algebra credits that will be accepted by any public postsecondary institution in Tennessee.

This document includes the following resources and instructional materials for the course:

LEARNING OBJECTIVES

Dual Credit College Algebra Competencies

- ☐ *Numeric and Algebraic Operations (23%)*
- ☐ *Describe Equations (5%)*
- ☐ *Solve Equations (23%)*
- ☐ *Solve Inequalities (10%)*
- ☐ *Function and their Properties (32%)*
- ☐ *Representation/modeling (7%)*

I. Numeric and Algebraic Operations (23%)

1) Factoring and Expanding Polynomials
   - Factor quadratics completely
   - Factor polynomials completely (degree ≤ 5)
   - Determine a binomial expansion
2) Operations with Numbers
   - Complex Numbers: Perform basic operations (add, subtract, multiply, divide, conjugate)
3) Operations with algebraic expressions
   - Perform basic operations (+, - ×, ÷) with rational expressions
   - Simplify complex rational expressions

4) Operations with exponents
- Apply the properties of exponents (including rational exponents)

5) Operations with logarithms
- Apply the properties of logarithms

II. Describe Equations (5%)

1) Write an equation of a line (parallel, perpendicular, point/slope, two points)
2) Write an equation of a parabola given vertex and one point.

III. Solve Equations* (23%)

1) Solve linear equations.
2) Solve application problems involving linear equations (mixture, motion, simple interest, constant rate job)
3) Graph linear equations in the Cartesian coordinate system.
4) Solve systems of linear equations (two equations with two unknowns)
5) Solve quadratic equations that have both real and complex solutions (factoring, quadratic formula, square root method)
6) Graph quadratic equations in the Cartesian coordinate system.
7) Solve absolute value equations (linear)
8) Solve rational equations
9) Solve radical equations involving a single square root
10) Solve exponential equations
11) Solve logarithmic equations

*One variable unless in Cartesian coordinate system

IV. Solve Inequalities* (10%)

1) Solve linear inequalities
2) Solve application problems involving linear inequalities
3) Solve quadratic inequalities
4) Solve absolute value inequalities
5) Graph linear inequalities in the Cartesian coordinate system.
6) Graph systems of linear inequalities in the Cartesian coordinate system (2 inequalities with 2 unknowns)

* One variable unless in Cartesian coordinate system

V. Function and Their Properties** (32%)

1) Definitions (Each test may contain a variety of functions including linear, polynomial (degree $\leq$ 5), rational, absolute value, power, exponential, logarithmic and piecewise- defined)

- Determine whether a relation is a function from its graph.
- Evaluate functions for given values.
- Determine type of functions (linear, quadratic, polynomial greater than 2nd degree, rational, exponential, logarithmic, radical, absolute value, piece-wise)
- Determine domain of a function from equation or graph.
- Determine range of a function from a graph.

2) Graphs and Their Properties (Graphing includes sketch of the graph showing intercepts, symmetry and other important characteristics)
- Graph polynomial functions of degree greater than 2.
- Graph exponential functions.
- Graph logarithmic functions.
- Graph rational functions (asymptotes – horizontal and vertical)
- Graph radical functions
- Identify intervals on which functions are increasing, decreasing and constant (from a graph)
- Identify and apply transformations to a graph (horizontal, vertical, reflections, stretching/shrinking)

3) Algebra of Functions and Inverse functions
- Perform basic function operations (add, subtract, multiply, divide)
- Evaluate composition of functions
- Simplify composite functions
- Determine if a given function has an inverse function
- Find the inverse function of a given function if it exists

VI. Representation/modeling (graphical, numerical, symbolic and verbal) (7%)
1) Solve real world problems involving variation, using both direct and inverse proportionality.
2) Solve real world problems involving exponential functions (compound interest, exponential growth and decay).
3) Solve real world problems involving logarithms (radioactive decay, decibels, or the Richter scale).

INSTRUCTIONAL TASKS

The following tasks can be used to support instruction in the statewide dual credit College Algebra (Advanced Algebra/Trigonometry) course.

1) **The Great Suit Case**

   A popular model of carry-on luggage has a length that is 10 inches greater than its depth. Airline regulations require that the sum of the length, width, and depth cannot exceed 40 inches.
   a) Write a function that models the volume of the carry-on luggage.
   b) If the volume of the luggage is 200 cubic inches, determine the depth. Explain your solution.
   c) Rewrite the function as an inequality. Explain the effect of the inequality sign on the size of the luggage.

2) **The Slugger**

   Suppose Bryce Harper threw a baseball from left field to second base.
   a) Write a formula that relates Bryce's height, initial velocity, and time. (Assume that the ball is caught at ground level.)
   b) Bryce Harper is 6'2" tall. If the maximum height of the baseball after 1.5 seconds is 30 feet high, write a formula that models the height at a given time k.
   c) After how many seconds was the baseball caught?

3) **Tree Trimmer**

   A tree trimming contractor is trying to decide if he should purchase a truck with a lift for $42,000. The vehicle requires an average expenditure of $6.50 per hour for fuel and maintenance. The operator is paid $11.50 per hour.
   a) Write a function for the total cost.
   b) Discuss the possible domain and range for the function you wrote in part a.
   c) Develop a formula for profit.
   d) Assuming that the contractor will charge $30 per hour, how long will it take to make a profit?
   e) The contractor also found a truck for sale with a price of $38,000 with an average expenditure of $7.50 per hour for fuel. Which truck should he purchase?

4) **The Homecoming Event**

   This year's homecoming game was a huge revenue boost for the high school. As a result of the large attendance, a grand total of $78,500 was raised: $65,000 from attendance and $15,500 from concessions. A total of 9,000 tickets were sold. Adult tickets cost $8 and student tickets cost $6.
   a) Write a model to accurately reflect the ticket sales information.
   b) How many adult tickets were sold?

c) Assume the number of tickets sold does not change and ½ of the adult tickets were pre-purchased for half price. How would this effect the revenue?

d) How would the pre-purchased tickets affect your model?

e) Describe another method to solve the problem.

f) Which method is the most efficient and why?

5) **Power Station**

A power station is on one side of a river that is 1 mile wide. A factory is 8 miles downstream on the other side of the river. It costs $24 per foot to run power lines over land and $30 per foot to run them underwater.

a) Write an equation for the total cost of running power lines to the factory in terms of the distance from the power station when the lines cross the river.

b) What will the total cost of running the power lines be if they cross the river 3 miles from the power station?

c) What will the total cost of running the power lines be if they cross the river 5 miles from the power station?

d) Extension: To the nearest mile, where should the power lines cross the river in order to minimize the cost?

6) **The Battery Solution**

While working at Everstart, a battery company, you have to fill a 55 gallon barrel with a battery acid solution that has a 75% concentration of sulfuric acid. All you have available is a solution that has a 100% concentration of sulfuric acid and a second solution that has a 40% concentration of sulfuric acid.

a) How many gallons of each type of sulfuric are needed to complete the job?

b) You accidentally added a total of 35 gallons of the 100% solution. How much more of the 40% solution must be added to achieve a 75% solution?

c) Because you now have more than 55 gallons of the 75% solution, you need to take the extra gallons and dilute them with water to make them environmentally safe. How many gallons of water are needed to have a 10% concentration for the extra gallons of acid solution?

7) **Duck Dynasty**

In Sy's "How to Make a Duck Call" class, you have two examinations with grades of 86 and 88. There is an optional final exam which counts as one grade. If you take the final, it will count. It cannot be excluded. Do you want to take the final?

a) What grade can you make on the final exam to receive an A?

b) Write an inequality to represent the grade range needed to make an A. Justify your answer.

c) What grade range on the final exam must you earn to maintain your B average?

d) Write an absolute value inequality to find the grade needed to maintain your B average.

e) Do you now want to attempt the final exam? Why or why not?

8) How It's Made

A steel drum in the shape of a right circular cylinder is required to have a volume of 100 ft^3.

a) How much material is required for a drum with a radius of 3 ft?

b) How much material is required to build a drum with a radius of 5 ft?

c) What is the smallest radius possible for the drum?

9) Nashville Night Lights

Merchants in downtown Nashville are going green. They are changing out their outdoor light bulbs to new ones that use less electricity. They are only replacing them as they burn out. On average about 18% of the old light bulbs burn out each month. Assuming there are about 400,000 light bulbs, how long will it take them to replace all the bulbs?

a) How could you write an equation to help you answer this? Include the equation and explain why it would work.

b) How long will it take for there to be 100,000 bulbs left to be changed? Explain how you know.

c) How long will it take for there to be 1,000 bulbs left to be changed?

d) How long will it take for there to be 1 bulb left to be changed?

e) How long will it take until all the bulbs are replaced?

f) Describe another method to solve this problem. Explain why it would also work.

g) Which method is the most efficient? Why?

10) Contest Winner

You entered a contest and WON! You can choose one of 2 prizes. The one you do not choose will go to the person in second place. Option #1: A brand new Ferrari valued at $300,000! Option #2: One penny on the first day, double that to two pennies on the second day, which would again be doubled to four pennies on the third day, and so on for a month (30 days). Which prize will you choose?

a) Which prize do you predict to be worth more money?

b) How many cents is $300,000?

c) How would you write your daily winnings for option #2 in exponential form? Include how you could express the amount received on day *n.* Explain how you came up with your answer and why it works.

d) Write an inequality based on your prediction in question 1. (Hint: Write your answer to question 2 in scientific notation.)

e) Test your prediction. Show how you are conducting your test.

f) Did you choose well? Or should you change your mind? Explain.

SAMPLE ASSESSMENT QUESTIONS

The following sample assessment questions can be used to support instruction in the statewide dual credit College Algebra (Advanced Algebra/Trigonometry) course. The questions are aligned with the course competencies.

I.   Numeric and Algebraic Operations

<u>Factoring and Expanding Polynomials</u>

Factor quadratics completely
1.   Factor completely: $x^2 + 7x - 44$

2.   Factor completely: $4x^2 - 49$

3.   Factor completely: $24x^2 - 6x - 9$

4.   Factor completely: $98x^2 - 18$

Factor polynomials completely (degree ≤5)
1.   Factor completely: $x^4 - 16$

2.   Factor completely: $x^3 - 4x^2 + 8x - 32$

3.   Factor completely: $2x^2 - xy - 6y^2$

Determine a binomial expansion
1.   Expand the binomial and express the result in simplified form: $(3x + 2)^5$

2.   Expand the binomial and express the result in simplified form: $(2a - 3b)^4$

<u>Operations with Numbers</u>

Complex Numbers: Perform basic operations (add, subtract, multiply, divide, conjugate)
1.   Simplify: $(3 - 2i) + (8 + 2i)$

2.   Simplify: $(4 + 2i) - (7 + 6i)$

3.   Simplify: $(3 + 2i)(6 - i)$

4.   Simplify: $(9 - 4i)^2$

5.   Simplify: $\frac{4}{2-i}$

6.   Simplify: $\frac{6+5i}{3+i}$

Operations with Algebraic Expressions

Perform basic operations (+, −, ×, ÷) with rational expressions

1. Simplify completely: $\dfrac{6}{x-6} + \dfrac{4}{x-2}$

2. Simplify completely: $\dfrac{4}{2x+1} - \dfrac{3}{x+2}$

3. Simplify completely: $\dfrac{3x+12}{7x-14} \cdot \dfrac{3x-6}{x^2-4x-32}$

4. Simplify completely: $\dfrac{4x^2-16}{3x-6} \div \dfrac{16x^2+48x+32}{9x+36}$

Simplify complex rational expressions

1. Simplify completely: $\dfrac{6-\frac{4}{x}}{3+\frac{5}{x}}$

Operations with Exponents

Apply the properties of exponents (including rational exponents)

1. Simplify: $\left(256x^8 y^{20}\right)^{\frac{1}{4}}$

2. Simplify: $\left(3x^{\frac{2}{3}}\right)\left(4x^{\frac{3}{4}}\right)$

3. Simplify: $\dfrac{36y^{\frac{1}{5}}}{18y^{\frac{3}{10}}}$.

Operations with Logarithms

Apply the properties of logarithms

1. Expand: $\log_4(256x^2)$.

2. Expand: $\log_3\left(\dfrac{a^8 b^5}{c^9}\right)$.

3. Expand: $\ln(e^5 r^3)$.

II.   Describe Equations

Write an equation of a line (parallel, perpendicular, point/slope, two points)

1. Find the equation of a line in slope-intercept form that has slope 6 and passes through $(-1, 3)$.

2. Find the equation of a line in slope-intercept form that passes through $(6, 2)$ and $(2, -1)$.

3. Find the equation of a line in slope-intercept form that passes through $(-2, -3)$ and is parallel to the line $4x + 5y = 2$.

4. Find the equation of a line in slope-intercept form that passes through $(-2, -3)$ and is perpendicular to the line $4x + 5y = 2$.

Write an equation of a parabola given vertex and one point

1.   Write an equation of a parabola that passes through the point $(2, 4)$ with vertex $(-1, -1)$ in vertex form.

III.   Solve Equations

Solve linear equations.

1.   Solve: $3x + 2(x - 7) = (9x - 1) - (x - 8)$.

Solve application problems involving linear equations (mixture, motion, simple interest, constant rate job)

1.   Mooncents Coffee Company is not selling its Lava Roast Coffee Beans, which normally are priced at \$6.00 per pound, or its Volcano Boost Coffee Beans, which normally are priced at \$9.00 per pound, very well. The manager decides to blend the two and create a new product—Volcano Lava Blend Coffee—that sells for \$8.25 per pound. How many pounds of Lava Roast Coffee Beans should be mixed with 100 pounds of Volcano Boost Coffee Beans to ensure no change in profit?

2.   A motorboat can maintain a constant speed of 40 miles per hour relative to the water. The boat makes a trip upstream in one hour and then returns to its starting point in 45 minutes. What is the speed of the water's current?

3. Brea earned a \$15,000 bonus at work and decided to invest it in two certificates of deposit (CDs), one which earned 2% interest in one year and one which earned 5% interest in one year. If Brea earned \$600 in interest at the end of the year, how much did she invest in each CD?

4. Working alone, Suzy can clean the fourth floor of Euclid Towers in 3 hours. Ralf can clean the fourth floor of Euclid Towers in 5 hours when he works by himself. How long will it take Suzy and Ralf to clean the fourth floor of Euclid Towers if they work together?

Graph linear equations in the Cartesian coordinate system

1. Graph: $y = \dfrac{-2}{3}x - 4$

Solve systems of linear equations (two equations with two unknowns)

1. Solve the system: $\begin{cases} 3x - 4y = 8 \\ 6x - 2y = 4 \end{cases}$

2. Solve the system: $\begin{cases} y = 3x - 2 \\ 6x - 2y = 4 \end{cases}$

3. Solve the system: $\begin{cases} 4x + 2y = 6 \\ 2x + y = 1 \end{cases}$

Solve quadratic equations that have both real and complex solutions (factoring, quadratic formula, square root method)

1. Solve the equation: $3x^2 - 2x + 4 = 0$

2. Solve the equation: $x^2 - 5x + 6 = 0$

3. Solve the equation: $(3x - 2)^2 = 16$

Graph quadratic equations in the Cartesian coordinate system

1. Graph $g(x) = (x - 1)^2 + 2$

2. Graph $h(x) = x^2 - 5x + 4$

Solve absolute value equations (linear)

1. Solve: $|6x + 1| - 4 = 12$

Solve rational equations

1. Solve: $\dfrac{4}{x-4} + \dfrac{9}{x+5} = \dfrac{3}{x^2+x-20}$

Solve radical equations involving a single square root

1. Solve: $x + \sqrt{20 - x} = 8$

Solve exponential equations
1.   Solve: $3^{4x+1} = 81$

Solve logarithmic equations
1.   Solve: $\log_2(x - 6) + \log_2 x = 4$

IV.   Solve Inequalities

Solve linear inequalities
1.   Solve: $7(2x - 3) \geq -3(3x + 4) - 1$

Solve application problems involving linear inequalities
1.   Joaquin's Portraits Company charges an initial sitting fee of $50 plus $15 per hour to make senior pictures. Ritzy Photography charges an initial sitting fee of $30 plus $20 per hour to make senior pictures. After how many hours will the charges be equal? If you anticipate it taking 8 hours to have your senior pictures taken, which company should you choose? If you anticipate it taking 2 hours To have your senior picture taken, which company should you choose?

Solve quadratic inequalities
1.   Solve: $6x^2 - x - 12 > 0$

2.   Solve: $3x^2 - x - 2 \leq 0$

Solve absolute value inequalities
1.   Solve: $|2x - 4| + 3 > 6$

2.   Solve: $|3 - x| - 1 \leq 2$

Graph linear inequalities in the Cartesian coordinate system
1.   Graph: $2x - 3y < 4$.

Graph systems of linear inequalities in the Cartesian coordinate system (2 inequalities with 2 unknowns)
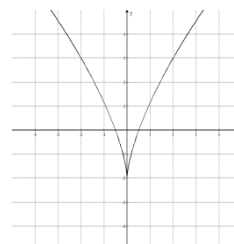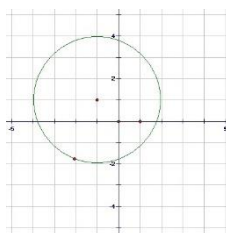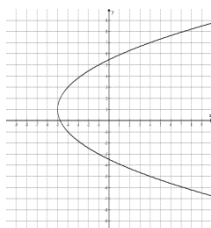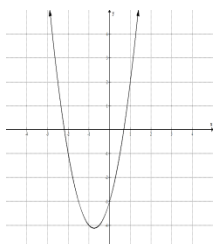1.   Graph: $\begin{cases} y > 3x + 2 \\ y \leq \frac{-1}{2}x + 3 \end{cases}$

2.   Graph: $\begin{cases} x - y < 5 \\ x - y \geq 3 \end{cases}$

V. Functions and their Properties

Determine whether a relation is a function from its graph

1. Determine whether or not each graph is a graph in which y is a function of x:

Evaluate functions for given values

1. Evaluate the function $f(x) = \sqrt{x^3 - 6x + 1}$ for $f(-2)$ and simplify.

2. Evaluate the function $f(x) = \frac{1}{2}(4)^x$ for $f(3)$ and simplify.

3. The function $P(x) = 2.5x + 15$ models the relationship between the number of hours a bicycle is rented and the profit the rental store makes. Find $P(6)$, the profit the store makes after renting the bicycle for 6 hours.

Determine type of functions

1. Identify the type of function: $f(x) = \dfrac{3x + 5}{x^2 - 2}$

2. Identify the type of function: $f(x) = x^4 - 2x^3$

3. Identify the type of function: $f(x) = 3^x$

4. Identify the type of function: $f(x) = 2\sqrt{x + 1} - 6$

5. Identify the type of function: $f(x) = \left|-9x + 4\right|$

6. Identify the type of function: $f(x) = 7x - 21$

7. Identify the type of function: $f(x) = \begin{cases} x^2; \ x \le -1 \\ x + 5; x > -1 \end{cases}$

8. Identify the type of function: $f(x) = \log_2 x$

9. Identify the type of function: $f(x) = x^2 + 10$

Determine domain of a function from an equation

1. Find the domain: $f(x) = \dfrac{2x-3}{x^2-4x}$

2. Find the domain: $f(x) = \sqrt{x+5}$

Determine domain and range of a function from a graph

1. Use the graph to determine the function's domain and range:
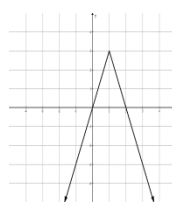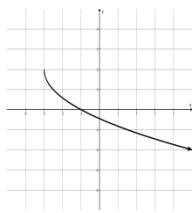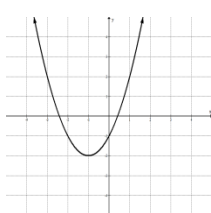

<u>Graphs and Their Properties</u>

Graph polynomial functions of degree greater than 2

1. Graph the function: $f(x) = x^3 + 2x^2 - 3x + 1$

2. Graph the function: $f(x) = 2 + 3^x$

3. Graph the function: $f(x) = \log 4x$

4. Graph the function: $f(x) = \dfrac{x^2-1}{x+2}$ Be sure to identify any horizontal or vertical asymptotes.

5. Graph the function: $f(x) = -\sqrt{x+1} - 3$

6.
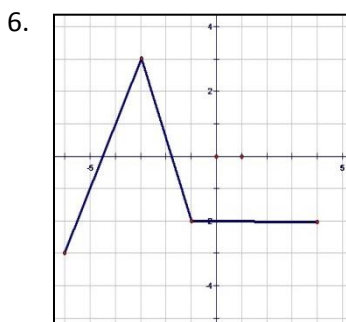
   a. Identify the interval where the function is increasing.

   b. Identify the interval where the function is decreasing.

   c. Identify the interval where the function is constant.

7.
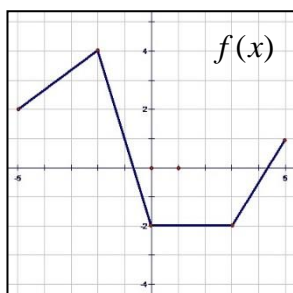
a. Use the graph of $y = f(x)$ to graph the function
   $g(x) = -f(x) + 3$

b. Use the graph of $y = f(x)$ to graph the function
   $g(x) = f(x+4) - 2$

c. Use the graph of $y = f(x)$ to graph the function
   $$g(x) = \frac{1}{3} f(x-1)$$

Algebra of Functions and Inverse Equations

Perform basic function operations

1. Given $f(x) = 2x^2 - 3x + 4$ and $g(x) = -x^2 + 3$, find $(f+g)(x)$.

2. Given $f(x) = 2x^2 - 3x + 4$ and $g(x) = -x^2 + 3$, find $(f-g)(x)$.

3. Given $f(x) = x^2 - 3x - 4$ and $g(x) = x - 4$, find $(fg)(x)$.

4. Given $f(x) = x^2 - 3x - 4$ and $g(x) = x - 4$, find $\left(\dfrac{f}{g}\right)(x)$.

Evaluate composition of functions

1. Given $h(x) = \dfrac{1}{3x(x-2)}$ find functions f and g so that $h(x) = (f \circ g)(x)$.

   a) $f(x) = \dfrac{1}{3x}$; $g(x) = \dfrac{1}{x-2}$ b) $f(x) = \dfrac{1}{3}$; $g(x) = x - 2$

   c) $f(x) = x^2 - 4$; $g(x) = \dfrac{x+2}{3x}$ d) $f(x) = \dfrac{x+2}{3x}$; $g(x) = x^2 - 4$

Simplify composition of functions

1. Given $f(x) = 7x - 2$ and $g(x) = 2x + 5$, find $(f \circ g)(x)$.

2. Given $f(x) = \dfrac{x}{x+4}$ and $g(x) = x^2 - 2$, find $(f \circ g)(x)$.

3. Given $f(x) = \sqrt{x^2 + 1}$ and $g(x) = x - 1$, find $(f \circ g)(x)$.

Determine if a given function has an inverse function

1. Does the graph represent a function that has an inverse function?
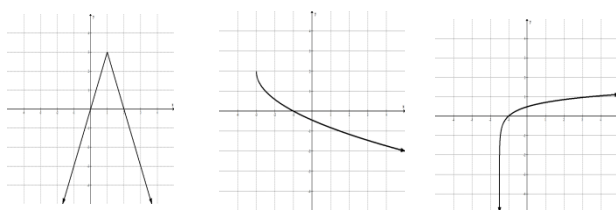
Find the inverse function of a given function if it exists

1. Find the inverse of the function if it exists: $f(x) = (x+2)^3$

2. Find the inverse of the function if it exists: $f(x) = \sqrt[3]{x-1}$

3. Find the inverse of the function if it exists: $f(x) = x^2 + 2x + 1$

VI.   Representation/modeling (graphical, numerical, symbolic and verbal)

Solve real world problems involving variation, using both direct and inverse proportionality

1. The amount of gas that a helicopter uses is directly proportional to the number of hours spent flying. The helicopter flies for 2 hours and uses 24 gallons of fuel. Find the number of gallons of fuel that the helicopter uses to fly for 6 hours.

2. When the temperature stays the same, the volume of a gas is inversely proportional to the pressure of the gas. If a balloon is filled with 36 cubic inches of a gas at a pressure of 14 pounds per square inch, find the new pressure of the gas if the volume us decreased to 12 cubic inches.

3. For a resistor in a direct current circuit that does not vary its resistance, the power that a resistor must dissipate is directly proportional to the square of the voltage across the resistor.

The resistor must dissipate $\frac{1}{16}$ watt of power when the voltage across the resistor is 6 volts.

Find the power that the resistor must dissipate when the voltage across it is 18 volts.

Solve real world problems involving exponential functions (compound interest, exponential growth and decay)

1. Margaret has $15,000 she would like to invest. She found a bank offering a CD that earns 1.15% interest when compounded annually. How much will her investment be worth in 5 years?

2. The population of a Tennessee city has an annual growth rate of 2.2%. If there are 42,000 people there now, how many will be living there in 12 years?

3. A 56m structure was built on bad soil. Each year the structure sinks into the ground 1.2cm. Assuming this rate remains constant, how far above the ground will the structure reach in 150 years?

Solve real world problems involving logarithms (radioactive decay, decibels, or the Richter scale)

1. In November 2012, an earthquake centered in eastern Kentucky registered a magnitude of 4.3 on the Richter scale. The Richter scale is used by seismologists to express the energy, or magnitude, of an earthquake. The formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$ expresses the magnitude, *M*, in terms of the energy released, *E*. Find the energy released by the earthquake in eastern Kentucky.